HIGH-ORDER NUMERICAL METHODS FOR WAVE PROPAGATION IN POROUS MEDIA

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Abstract

Groundwater is a hugely important resource in both developed and developing countries and makes up over 90% of the world's freshwater. The idea behind our work is to use wave scattering in saturated porous media to quantify aquifer state and hydraulic properties. We initially focused on solving related inverse problem using the Specfem software to model coupled elastic/poroelastic waves. Of particular importance to us is the accurate modelling of the dissipative properties of poroelastic waves. This lead us to develop high-order discontinuous Galerkin methods. In this paper, we present results for a two dimensional case.

1 INTRODUCTION

Groundwater is a fundamental natural resource that accounts for more than 90% of the planet's freshwater excluding glaciers. Traditional methods of aquifer (geological formations that contain water) exploration usually reveal very limited information about groundwater resources and generally do not permit accurate monitoring due to the immense computational challenges of interrogating large and complex data sets. This research is about building tools that permit accurate quantification of groundwater resources by making the maximum use of data, answering basic questions like what is actually there, how much can be extracted and how fast. For background on groundwater see [1].

The potential for using passive seismic data to map and characterise aquifers was considered in [2, 3]. Here, the seismic waves caused by frequent small magnitude earthquakes propagate through aquifers. In brief, elastic and poroelastic media scatter waves differently, essentially due to a disparity in wave speeds. Therefore, it is in principle possible to recover the water table and geometry of the aquifer from seismic data. While groundwater responses and changes in aquifer formation due to large earthquakes have been much studied, see for example [4] and the references cited therein, the potential for using small magnitude earthquakes to characterise aquifer properties appears to be relatively unexplored, [5].

In this paper, we consider a two dimensional model problem where seismic wave propagation is studied in a geometry consisting of a two layered porous aquifer and the elastic bedrock layer beneath it. In the model, spatial derivatives are approximated using the discontinuous Galerkin method while the time derivatives are approximated using the low-storage Runge-Kutta method [6].

2 DISCONTINUOUS GALERKIN METHOD

A promising approach for accurately approximating wave fields, for example, in poroelastic and elastic medium is the discontinuous Galerkin (DG) method [7, 8, 6, 9, 10]. The DG method is well-suited for large scale wave simulations. For example, the method can be effectively parallelized using the full power of super computer facilities or standard multicore computers. On the other hand, the DG method can handle complex geometries and since it is conservative large discontinuities within the material parameters. All previously mentioned topics are essential features for the method used in large scale wave models. Problems considered in this work contain large contrasts within the material parameters (e.g. wave speed values between soil and rock). Also in all cases, the computational domain is large (with respect to the wavelength) and therefore parallel computation is necessary in order to be able to compute results in acceptable computational time. For the seismic problems, realistic geometric scale for aquifers is easily tens of kilometers in each spatial direction.

3 NUMERICAL EXPERIMENT

In this numerical experiment, we consider a model that consists of two poroelastic [11, 12] subdomains and one elastic subdomain. The upper poroelastic layer is air saturated porous material while the lower one is water saturated. The idea of this model is to demonstrate how the current method can be applied to estimate aquifer properties, such as water table level and aquifer basement. Below the aquifer, we assume very low porosity and hence the material is considered to be purely elastic. In the aquifer, the water table level is assumed to be at depth 25 m and the basement at depth 50 m. Other dimensions of the model are assumed to be 2.5 km in length and 1 km in depth. On the top surface, we use the free boundary condition [13] while all other external boundaries are modelled as absorbing.

Figure 1 visualises the computational grid used in this experiment. On top left we show the closeup that is highlighted by white line in the bottom left picture. In our model, non-uniform basis orders are used. The basis order is selected separately for each triangular element of the computational grid. The selection formula for the basis order is based on the theory studied in [14] and which was further developed in [15, 16]. In practise, we set

$$p_\ell = \frac{2\pi a h^\ell_{\max}}{\lambda_\ell} + b, \tag{1}$$

where wavelength $\lambda_\ell = c^\ell_{\min}/f$, and p_ℓ is the order of the basis function in the ℓth element. Furthermore, f denotes the dominant frequency of the seismic source, $h^\ell_{\max}$ is the maximum distance between two nodes and $c^\ell_{\min}$ the smallest velocity component in the ℓth element of the computational grid. Parameters a and b control the local accuracy on each element. In this simulation we choose $[a,\ b] = [1.2768,\ 1.4384]$ near the seismic source, otherwise $[a,\ b] = [1.0294,\ 0.7857]$. These parameter choices means in practise that more accurate solution is demanded near the source, while the accuracy deceases in other parts of the model. Values for parameters a and b are taken from [15].

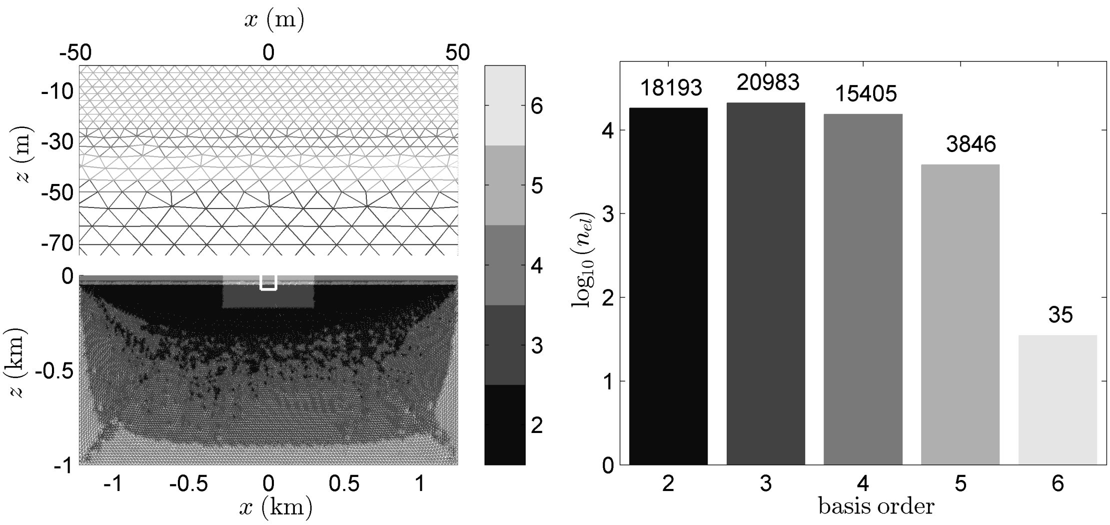

Figure 1: Discretization of the computational domain Ω via irregularly refined triangular mesh. Colorbar shows the order of the basis functions chosen on each element. The area marked by the white rectangle shown in the bottom left is shown in top left picture. Right: The number of elements as a function of basis order.

Figure 2 shows the velocity component $\sqrt{u^2 + v^2}$, where u denotes the horizontal and v the vertical solid velocity component, for four time instants. On each snapshot, the title shows the corresponding time instant. In this simulation, the source location is 1 m beneath the ground-air surface. As the seismic source, we use the exploding type of seismic moment tensor with a Ricker wavelet having 40 Hz dominant frequency f. Snapshots show the evolution of the wave structure inside the aquifer.

4 CONCLUSIONS

This work is part of the larger entity where the aim is to study what seismic signals reveal new information about aquifers. Our previous studies [2, 3] led us to develop the discontinuous Galerkin (DG) method to solve the corresponding forward problem (wave propagation from the seismic source to receiver). In brief, the DG method is well suited to study wave propagation in complex geometries and physical models.

In this paper, we considered an example where the seismic source was located close to the air-ground surface (i.e. active seismic prospecting). With the current DG scheme, the polynomial basis orders are chosen individually for each element of the computational grid. The idea of using the non-uniform basis orders is to

a) control the spatial accuracy $\Leftrightarrow$ in principle, higher basis orders lead to more accurate solution

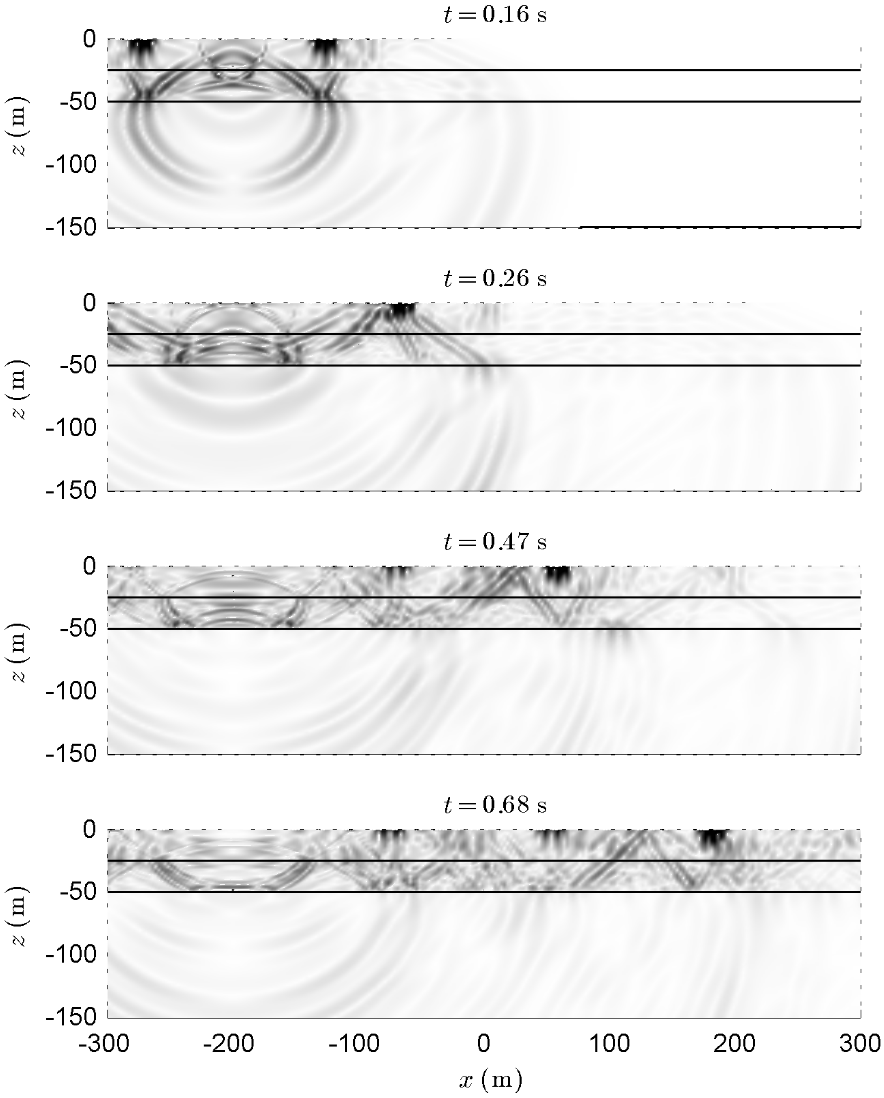

Figure 2: Velocity component $\sqrt{u^2 + v^2}$ near the exploding type of seismic source. Title shows the corresponding time instant.

b) reduce the overall computational demand $\Leftrightarrow$ accurate solution is demanded only in the region of interest, which can be the line between the source and receiver

The next step in our work is to use the same inversion strategy that we used in studies [2, 3] with the DG scheme discussed in this paper.

REFERENCES

[1] J. Bear. *Hydraulics of Groundwater*. Dover, 1979.

[2] T. Lähivaara, N.F. Dudley Ward, T. Huttunen, J. Koponen, and J.P. Kaipio. Estimation of aquifer dimensions from passive seismic signals with approximate wave propagation models. *Inverse Problems*, 30(1):015003, 2014.

[3] T. Lähivaara, N.F. Dudley Ward, T. Huttunen, Z. Rawlinson, and J.P. Kaipio. Estimation of aquifer dimensions from passive seismic signals in the presence of material and source uncertainties. *Geophysical Journal International*, 200: 1662–1675, 2015.

[4] C-Y. Wang and M. Manga. *Earthquakes and Water*. Springer, 2009.

[5] S.R. Pride. *Hydrogeophysics*, chapter Relationships between Seismic and Hydrological properties in Stochastic Hydrogeology. Springer, 2005.

[6] J.S. Hesthaven and T. Warburton. *Nodal Discontinuous Galerkin Methods: Algorithms, Analysis, and Applications*. Springer, 2007.

[7] W. Reed and T. Hill. *Triangular mesh methods for the neutron transport equation*. Technical report, Los Alamos National Laboratory, USA, 1973.

[8] B. Cockburn, G.E. Karniadakis, and C-W. Shu, editors. *Discontinuous Galerkin Methods: Theory, Computation and Applications*. Springer-Verlag, 2000.

[9] L.C. Wilcox, G. Stadler, C. Burstedde, and O. Ghattas. A high-order discontinuous Galerkin method for wave propagation through coupled elastic-acoustic media. *Journal of Computational Physics*, 229(24):9373–9396, 2010.

[10] N.F. Dudley Ward, T. Lähivaara, and S. Eveson. Discontinuous Galerkin finite element method for mechanical wave propagation in porous dissipative media. 2015. In preparation.

[11] M.A. Biot. Theory of propagation of elastic waves in a fluid saturated porous solid. I. Low frequency range. *Journal of the Acoustical Society of America*, 28(2): 168–178, 1956.

[12] M.A. Biot. Theory of propagation of elastic waves in a fluid saturated porous solid. II. Higher frequency range. *Journal of the Acoustical Society of America*, 28(2): 179–191, 1956.

[13] J.M. Carcione. *Wave Fields in Real Media: Wave propagation in anisotropic, anelastic and porous media*. Elsevier, 2007.

[14] M. Ainsworth. Dispersive and dissipative behaviour of high order discontinuous Galerkin finite element methods. *Journal of Computational Physics*, 198(1):106–130, 2004.

[15] T. Lähivaara and T. Huttunen. A non-uniform basis order for the discontinuous Galerkin method of the acoustic and elastic wave equations. *Applied Numerical Mathematics*, 61:473–486, 2011.

[16] T. Lähivaara and T. Huttunen. A non-uniform basis order for the discontinuous Galerkin method of the 3D dissipative wave equation with perfectly matched layer. *Journal of Computational Physics*, 229:5144–5160, 2010.