MODELLING THE DIRECTIONAL CHARACTERISTICS OF SOUND REFLECTIONS

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ABSTRACT

Sound reflections are usually modelled either as specular reflections or as diffusely scattered reflections (Lambert's cosine law). In the Odeon room acoustic model a weighted combination of the two models has been used. In the present study is derived a new model that takes into account the wavelength and the finite area of a rectangular reflecting surface. The model is based on sound transmission theory and the use of Babinet's principle that describes the equivalence between transmission and reflection. As could be expected, the result depends very much on the dimension of the reflecting surface relative to the wavelength. In stead of the solution for a single frequency, a simple approximation has been found for the case of an octave band.

1. INTRODUCTION

In the Odeon room acoustic computer model the sound reflections are currently simulated by a weighted combination of the two simple reflection models for specular reflection and diffuse reflection [1], see Fig.1. Snell's law for the specular reflection states that angle of reflection is equal to the angle of incidence, $\theta_r = \theta_i$. Lambert's law for the diffuse reflection states that the directional distribution of the reflected intensity is proportional to cos θ_r.. In the following the angle α relative to the plane of the surface will be used instead of θ_r.

$$\frac{I_r}{I_{r,\max}} = \cos(\theta_r) = \cos(\pi - \alpha) = \sin(\alpha) \qquad (1)$$

Both reflection models are based on the assumption of a very short wavelength compared to the dimensions of the reflecting surface. However, the reflection of a sound wave from a surface of finite size can never be perfectly specular due to the wavelength of the sound. In the following is derived a model for this reflection with special reference to the application in a ray tracing model.

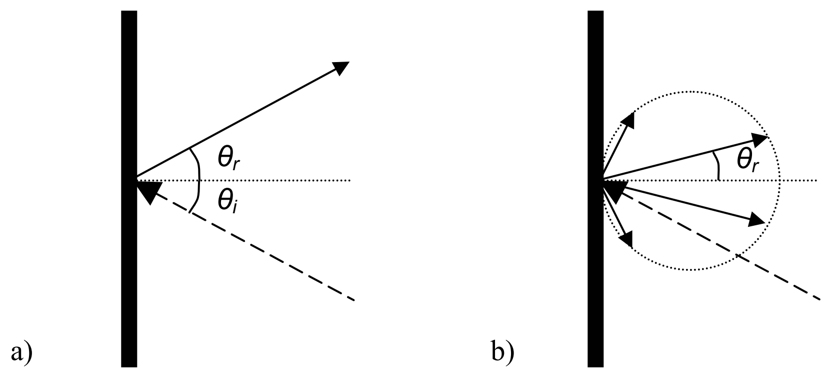

Figure 1. Reflection models (asymptotic high-frequency models), a) Snell's law for specular reflection, b) Lambert's law for diffuse reflection

2. BABINET'S PRINCIPLE

Babinet's principle states that the reflection from a plane rigid surface is equivalent to the transmission through an opening of the same geometrical shape surrounded by an infinite rigid baffle, see Fig. 2. The advantage of this principle in the present case is that a known solution to the problem of sound transmission through an opening can be applied to the equivalent problem of sound reflection from a surface of finite area.

Figure 2. *An example of Babinet's principle, a) reflection from a surface, b) transmission through an opening*

It is assumed that the incident sound is a plane wave and the surface is rectangular with dimensions $2a \cdot 2b$. The wavenumber is $k = 2\pi f / c$, where f is the frequency and c is the speed of sound. The direction of the incident sound is defined by the angles α_0 and β_0 relative to the x- and y-axis, see Fig. 3. Similarly, the direction towards the receiver point is defined by the angles α and β. The specular reflection will be in the direction ($\alpha = \alpha_0$, $\beta = \beta_0$). Using the result for sound transmission through a rectangular opening as derived in [2], the intensity of the reflected sound relative to the maximum value is:

$$\frac{I_r}{I_{r,\max}} = \left(\frac{\sin X}{X} \cdot \frac{\sin Y}{Y} \right)^2 \tag{2}$$

where

$$\begin{aligned} X &= ka(\cos\alpha - \cos\alpha_0) \\ Y &= kb(\cos\beta - \cos\beta_0) \end{aligned} \tag{3}$$

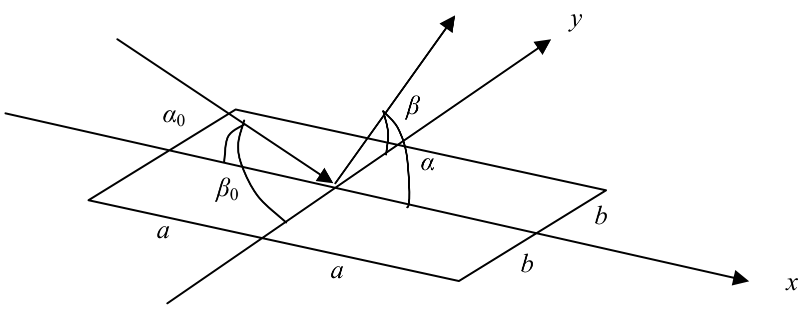

Figure 3. *Definition of angles of incidence and reflection from a rectangular surface*

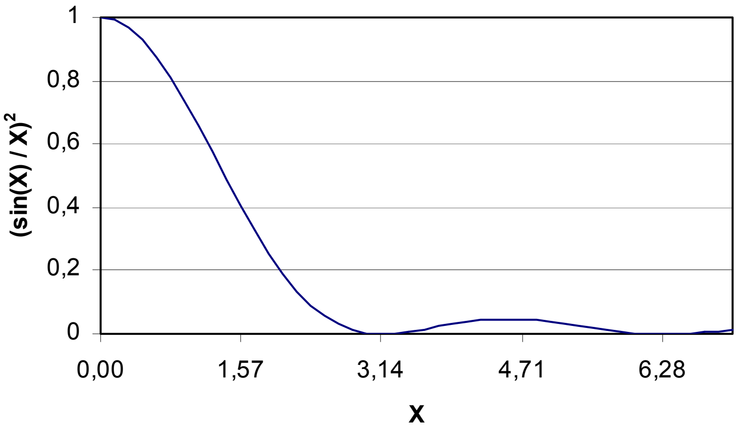

Figure 4. *The radiation function (sin (X)/X)^2 for a single frequency*

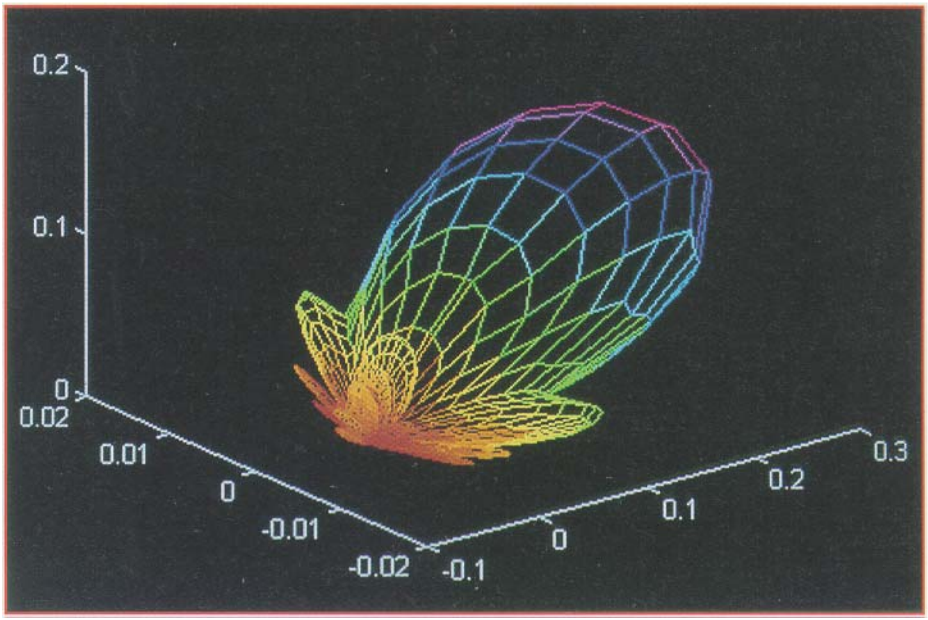

Figure 5. *Measured radiation pattern from a plane surface, after Kleiner [3]*

The radiation function $(\sin X / X)^2$ is shown in Fig. 4. It is noted that there are some side lobes, which are also seen in the measured result Fig. 5.

3. APPROXIMATION VALID FOR AN OCTAVE BAND

The radiation function $(\sin X / X)^2$ is valid for a single frequency, but in a room acoustic model it is more relevant to consider an octave band represented by the centre frequency. Averaging the radiated sound intensity in an octave band leads to the curve labelled octave band in Fig. 7. The radiation function for the centre frequency is not a bad approximation, but it has some drawbacks: For $X > 2\pi$ the function has much stronger fluctuations than the octave band average and for $X = 0$ the function has a singularity, which needs special consideration in a calculation.

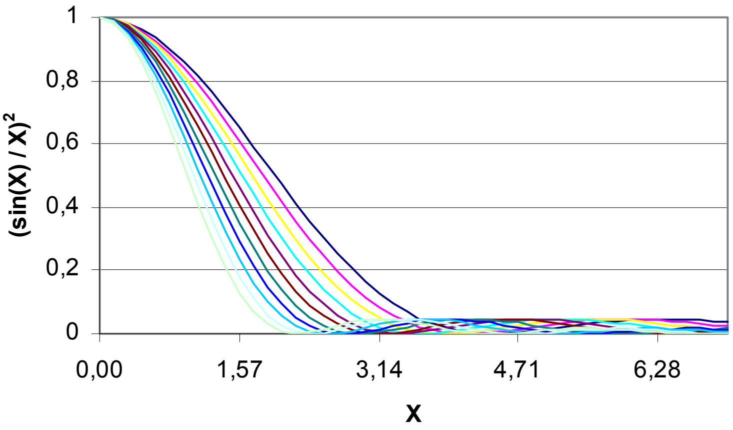

Figure 6. *The radiation function (sin (X)/X)^2 for 11 frequencies covering an octave band*

An approximation to the octave band average that does not have the same drawbacks has been found:

$$\left\langle \left(\frac{\sin X}{X} \right)^2 \right\rangle \approx \left(\cosh(0.67 \cdot X) \right)^{-2} \tag{4}$$

This function is shown in Fig. 7. The difference between the octave band average and this approximation is shown in Fig. 8 and it is seen that the maximum deviation is very small, around 0.03. It is noted that the side lopes of the radiation function have disappeared when the octave band average is considered. The constant 0.67 is found to be the optimum value in order to minimize the deviations from the true octave band average.

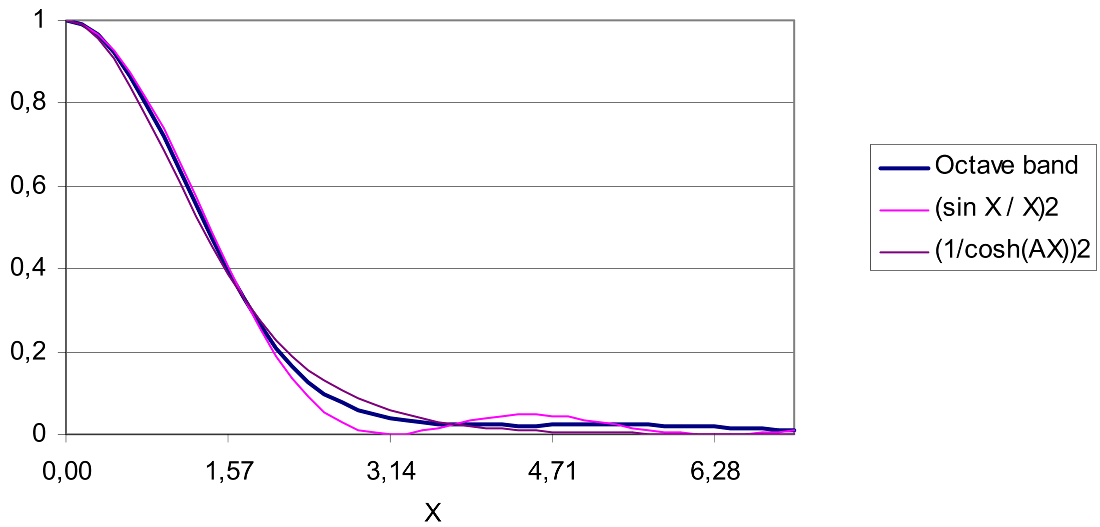

Figure 7. *The radiation functions for an octave band and for the centre frequency as a single frequency. The suggested approximation with the constant A = 0.67 is also shown.*

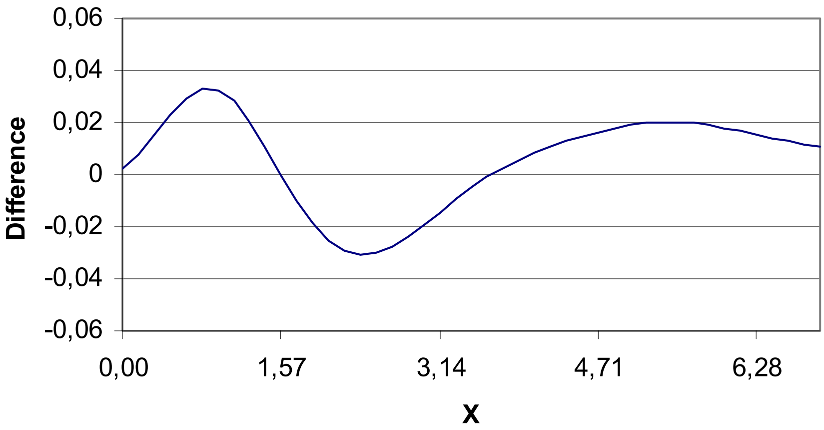

Figure 8. *The difference between the octave band average and the suggested approximation with the constant A = 0.67.*

4. EXAMPLES OF THE RADIATION FUNCTION

In the following Fig. 9 – 10 are displayed examples of the suggested approximate directivity function for different values of the parameter *ka*. The chosen angle of incidence α_0 is 90° and 30°, respectively. However, all angles of incidence including the grazing 0° are allowed in the model.

In addition to the directivity functions of the new reflection model is also shown the Lambert's law of diffuse reflection for comparison.

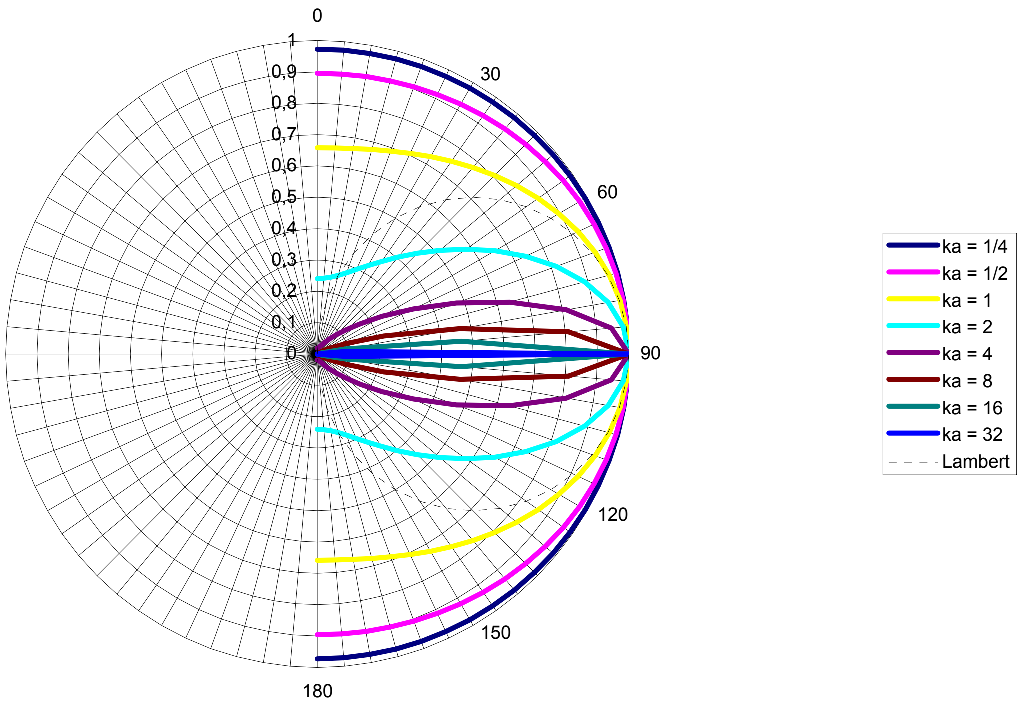

Figure 9. *The directivity functions for the angle of incidence α_0 = 90°.*

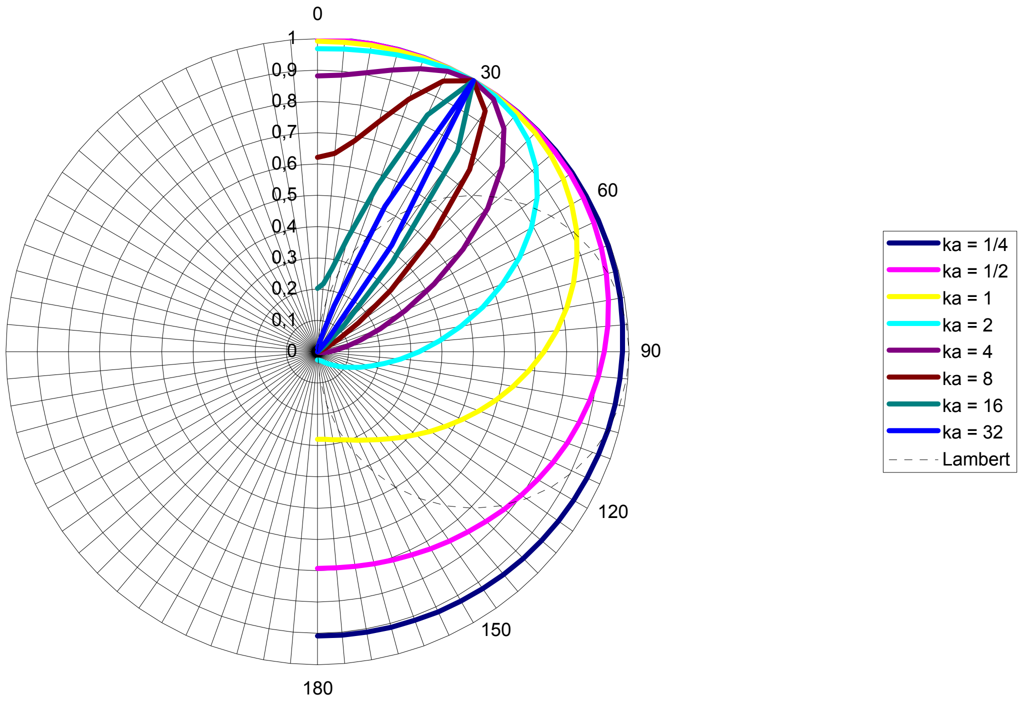

Figure 10. *The directivity functions for the angle of incidence α_0 = 30°.*

From the examples in Figures 9 and 10 is seen that the reflections are highly directive in the direction of the specular reflection for the high *ka* values, whereas the reflections are radiated with a nearly uniform directivity for the low *ka* values. The Lambert model is not particularly close to any of the examples. To give an idea of the surface dimensions and frequencies related to the various *ka* values, see Table 1.

2a	*ka = 1/4*	*ka = 1/2*	*Ka = 1*	*ka = 2*	*ka = 4*	*ka = 8*	*ka = 16*	*ka = 32*
0,22 m	125	250	500	1000	2000	4000	8000	
0,44 m	63	125	250	500	1000	2000	4000	8000
0,88 m		63	125	250	500	1000	2000	4000
1,75 m			63	125	250	500	1000	2000
3,50 m				63	125	250	500	1000

Table 1. *The relation between ka values and octave band centre frequencies, 2a is the dimension of the reflecting surface*

5. APPLICATION TO A RAY TRACING MODEL

In the Odeon room acoustic model the reflection points of the rays are used as secondary sources that radiate to the receiver point with a certain directivity pattern. If the new reflection model should be applied to such a ray tracing model it is necessary to consider the fact that there is a limited number of rays available, and this may cause a problem for high *ka* values when the reflections are highly directive. In Fig. 11 is shown the radiation to a receiver point from two reflection points with the distance *d*.

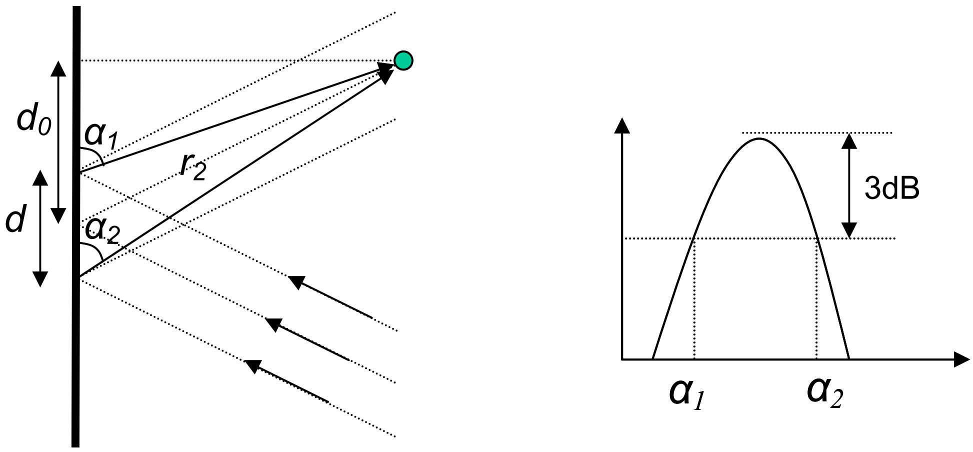

Figure 11. *Left: Two rays reflected to a receiver point. Right: The 3 dB bandwidth of the directivity function*

The distance d between two neighbour rays depends of the total number of rays N and the distance r_1 the rays have travelled from the source point:

$$d \approx \sqrt{\frac{4\pi r_1^2}{N}} = 2r_1\sqrt{\frac{\pi}{N}} \tag{5}$$

In order to avoid a gap between the neighbouring contributions it is suggested that the two reflections should lay within the 3 dB bandwidth of the directivity pattern, i.e. within the reflection angles α_1 and α_2, see Fig. 11. With the directivity function (4) the corresponding values of X_1 and X_2 are:

$$\cosh\left(0.67\cdot\left|X_{1,2}\right|\right) = \sqrt{2} \quad \Rightarrow \quad X_{1,2} = \pm 1.3155 \tag{6}$$

and thus the angles α_1 and α_2 can be determined from:

$$X_{1,2} = ka(\cos\alpha_{1,2} - \cos\alpha_0) \quad \Rightarrow \quad \cos\alpha_{1,2} = \cos\alpha_0 \pm \frac{1.3155}{ka} \tag{7}$$

The angles α_1 and α_2 can also be expressed by the distances d and r_2:

$$\cos\alpha_2 - \cos\alpha_1 \quad \cong \quad d / r_2 \quad = \quad 2\frac{r_1}{r_2}\sqrt{\frac{N}{\pi}} \quad = \quad 2\frac{1.3155}{ka} \tag{8}$$

It is concluded from this that the directivity function (4) can be used in a ray tracing model below a certain frequency, which depends on the distance to source and receiver, number of rays and the size of the reflecting surface:

$$(ka)_{\max} \quad \approx \quad 1{,}3155 \cdot \frac{r_2}{r_1}\sqrt{\frac{N}{\pi}} \tag{9}$$

If the number of rays N is not sufficient to allow the directivity function to be used at all frequencies of interest, the pragmatic solution could be to lock the directivity function at the max ka value, and apply this at the highest frequencies.

6. CONCLUSIONS

The reflection from finite size surfaces has been analyzed and a simple approximation to the octave band averaged directivity pattern has been derived. At high frequencies and large surfaces the directivity is very sharp in the direction of the specular reflection, but at low frequencies and small surfaces the directivity tends to be uniform. The Lambert cosine law is not a particularly good approximation.

7. REFERENCES

[1] Rindel, J.H. “Diffusion of Sound in Rooms - An Overview.” 15th ICA, Proceedings vol. 2, 633-636. Trondheim, 1995.

[2] Rindel, J.H. “Transmission of Traffic Noise through Windows - Influence of Incident Angle on Sound Insulation in Theory and Experiment”. Report No. 9, Laboratoriet for Akustik, DTH, Lyngby, 1975.

[3] Kleiner, M. “Ojämna väggar ger bättre ljud“, Bygg & teknik 88:1, 22-24, 1996.