FULL-WAVE MODEL FOR A LOUDSPEAKER

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1 INTRODUCTION

The modeling of the acoustic wave fields often provides additional information for acoustical measurements. In the case of long wavelengths, wave problems can be modeled using full-wave type methods which include the finite element, finite difference and boundary element methods. Unfortunately, when the wavelength decreases, these traditional full-wave modeling techniques become increasingly expensive since a sufficient number of discretization points per wavelength (10 points per wavelength is considered as the rule of thumb) is required to obtain a reliable solution. In addition, the numerical pollution due to the accumulation of phase error forces the use of even more grid points per wavelength to keep the relative error of the solutions sufficiently low.

A promising candidate for solving the wave propagation problems with a reduced computational cost is the Ultra Weak Variational Formulation (UWVF) [1]. This approach uses same computation meshes as the standard finite element method. However, the idea of the UWVF approach is that the sound field can be approximated locally using plane wave basis functions. In most cases, the use of the plane wave basis significantly reduces the CPU-time and the need of memory compared to conventional techniques (such as the finite element method).

In this study, the UWVF method is used for approximating the time harmonic wave propagation in three spatial dimensions (3D). We consider geometry which contains a real loudspeaker in free space. Results are computed using the MPI parallelized FORTRAN90 UWVF solver code. The used PC-cluster contains 24 Pentium 4 with 48 GB total RAM. The main goal of this work is to study the directivity of the used loudspeaker.

2 THE HELMHOLTZ EQUATION

In this study, the full-wave solution means the numerical solution of the acoustical wave equation in the frequency domain (the Helmholtz equation). In this section, the UWVF for the inhomogeneous Helmholtz problem is outlined shortly. The truncation of an unbounded problem is done using the perfectly matched layer (PML) [2], which is the numerical damping layer surrounding the computational domain. A more thorough presentation of the coupled UWVF-PML method can be found in [3].

Next, we formulate the acoustic wave problem. Let us assume that Ω is a 3D domain having the boundary Γ and that the acoustical pressure field P can be written in following form

$$P(r,t) = p(r)e^{-i\omega t}, \tag{1}$$

where $r = (x, y, z)$ is the spatial variable, t is the time, p denotes the pressure field (no time dependence) and ω is the angular frequency $(\omega = 2\pi f)$. Under these assumptions the Helmholtz equation with boundary condition(s) can be written as

$$\nabla \cdot \left(\frac{1}{\rho} \nabla p \right) + \frac{\kappa^2}{\rho} p = g_\Omega \quad \text{in } \Omega, \tag{2}$$

$$\left(\frac{1}{\rho} \frac{\partial p}{\partial n} - i\sigma p \right) = Q \left(-\frac{1}{\rho} \frac{\partial p}{\partial n} - i\sigma p \right) + g \quad \text{on } \Gamma, \tag{3}$$

where $\kappa = \omega/c + i\alpha$ is the wave number, ρ is the density and g_Ω stands for a volume wave source. On the boundary the parameters $Q \in C$ with $|Q| \leq 1$ and σ are real and positive. Finally, the complex valued source function on the exterior boundary Γ is denoted by g.

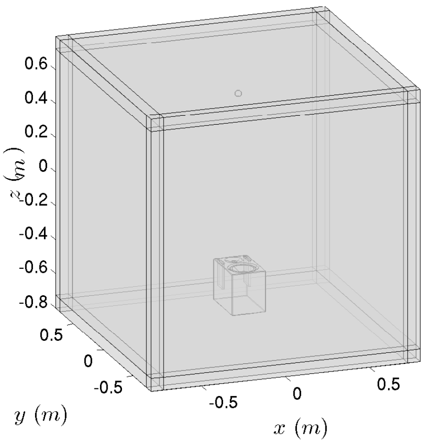
For the UWVF, the domain Ω is partitioned into a collection of finite elements (tetrahedron is a natural choice in 3D). After the mesh partitioning, the weak formulation can be applied individually for each element. Furthermore, the communication between adjacent elements is handled using the numerical flux. The final weak form for the problem is obtained by summing over all of the elements. More detailed derivation of the weak form can be found in [4].

3 NUMERICAL EXPERIMENTS

In our numerical experiments, we study the wave propagation in homogeneous medium. In particular, our focus in the following simulations is to study directivity of the loudspeaker by computing the pressure distribution in the far-field. In all of the simulations the wave propagation is studied in air. More precisely the speed of sound c and the density ρ are 340 m/s and 1.2 kg/m^3, respectively. Attenuation is ignored in all of the cases. In this study two frequencies for wave source are used. For low frequency simulations we use 1000 Hz and for high frequency 5000 Hz.

Before calculations we need to generate the problem geometry. The problem geometry consists of the loudspeaker and the surrounding free space. First, we generate the loudspeaker geometry from its CAD geometry file using the Gambit software. Then, the Comsol Multiphysics software was used to make the free space geometry (box with PML region). After that, we unite the loudspeaker geometry in the box. For the UWVF simulation, the geometry of the problem needs to be partitioned into elements i.e. computational mesh. For that purpose the Comsol Multiphysics software was used.

Figure 1 shows the problem geometry. In Figure 1 the volume between the inner and the outer cube is the PML region. The PML thickness for the discant simulations is 7.5 cm and 69 cm for the bass simulations. The whole computation domain for low frequencies is $\Omega = [-1.725,\ 1.725]^3$ m and $\Omega = [-0.8,\ 0.8]^3$ m for high frequencies.



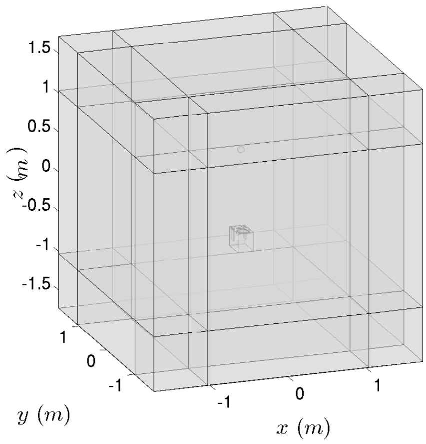


Figure 1: *Problem geometry for high frequencies (Left) and low frequencies (Right).*

The source is introduced to the model by using an inhomogeneous Neumann boundary condition on the discant or the bass surface depending on the frequency. Other parts of the loudspeaker are handled using the sound-hard boundary condition, except the end of the reflex tube, which contains the absorbing boundary condition. Finally, the remaining surfaces of the geometry contain the absorbing boundary condition.

Before going any further, stability indicator for the solutions must be introduced. In this study we only want to check the grid density. The indicator (rule of thumb) of the grid density presents, in practice, how many approximation points there are per wavelength. This can be presented as

$$N = \frac{\lambda}{h_{\max}}, \tag{4}$$

where λ is the wavelength, $h_{\max}$ is the longest distance between two nodes of the adjacent element in the computation mesh and N is the number of points per wavelength.

The computation meshes are shown in Figure 2. In Figure 2 the cross-section of the whole meshes and also the surface meshes of the loudspeaker are shown. Note that the source surface is shown in different color in the surface meshes. Computation meshes consists of 761281 ($h_{\max} = 0.0714$) elements for high frequency simulations and 735644 ($h_{\max} = 0.0963$) elements for low frequency simulations. For bass element simulations frequency of 1000 Hz is used, from which we obtain the grid density $N \approx 3.53$ (4). Similarly for high frequency simulations we use the frequency of 5000 Hz from which we get $N \approx 0.95$.

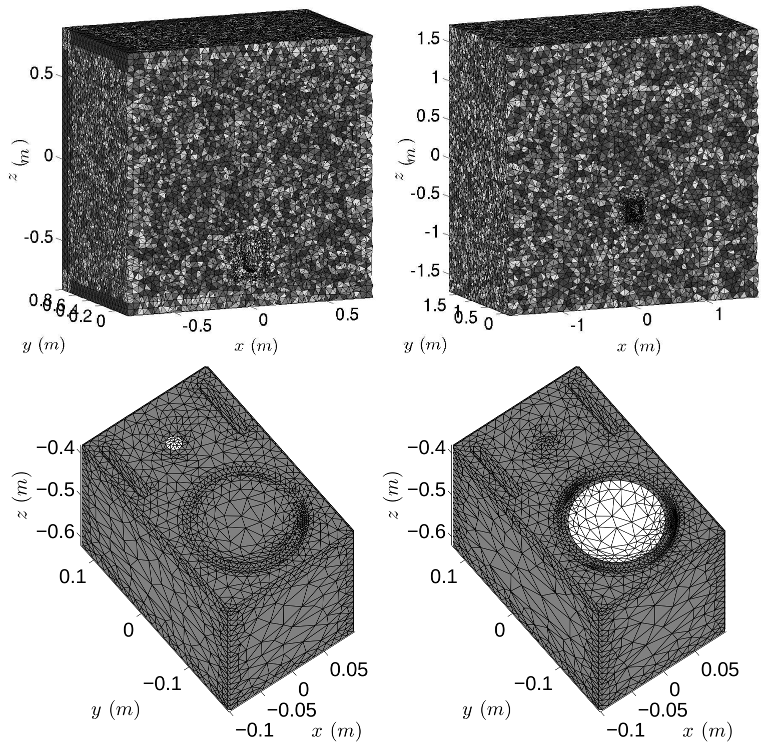


Figure 2: *Meshes used in the computations. Left column: For 5000 Hz. Right column: For 1000 Hz. Top row: Whole element partitions. Bottom row: The surface meshes of the loudspeaker.*

The farfield directivity patterns at frequencies 1000 and 5000 Hz are shown in Figure 3. As can be seen from Figures 3(a) and 3(c), both frequencies produces symmetric shapes in plane xz. In plane yz it is interesting to notice how the non-active speaker effects on the shape of the farfield. The interference behind the loudspeaker (xz and yz planes) can also be seen from the farfield directivity patterns.

Finally, Figure 4 shows the pressure fields. In these snapshots the surface of the loudspeaker is also included. Figure 4 shows the dB level and real-part of the solutions at a single frequency.

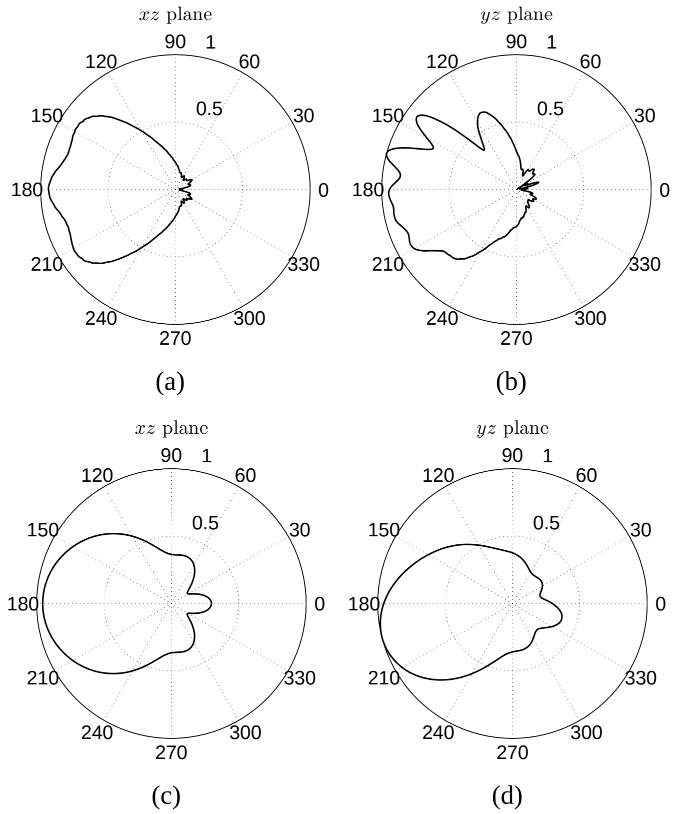


Figure 3: *Top row: Farfields at 5000 Hz. Bottom row: Farfields at 1000 Hz. The title shows the studied plane.*

4 CONCLUSIONS

In this work the Helmholtz equation is solved in 3D using the UWVF method. The main goal of the work was to study directivity of the loudspeaker. The geometry of the problem contained the real geometry of the loudspeaker which was located into a free space. Solutions were computed in low (1000 Hz) and high (5000 Hz) frequencies using the parallelized UWVF solver.

While the results show that simulation of the loudspeaker is possible, a detailed comparison with measurements still needs to be made to validate the simulation accuracy. On the other hand, the used model can be further improved. For example, one must note that our model does not approximate the reflex tubes correctly. This is handled only by using the absorbing boundary condition at the end of the tube. The model for loudspeaker surface would also be better by using the real surface impedance of the used material.

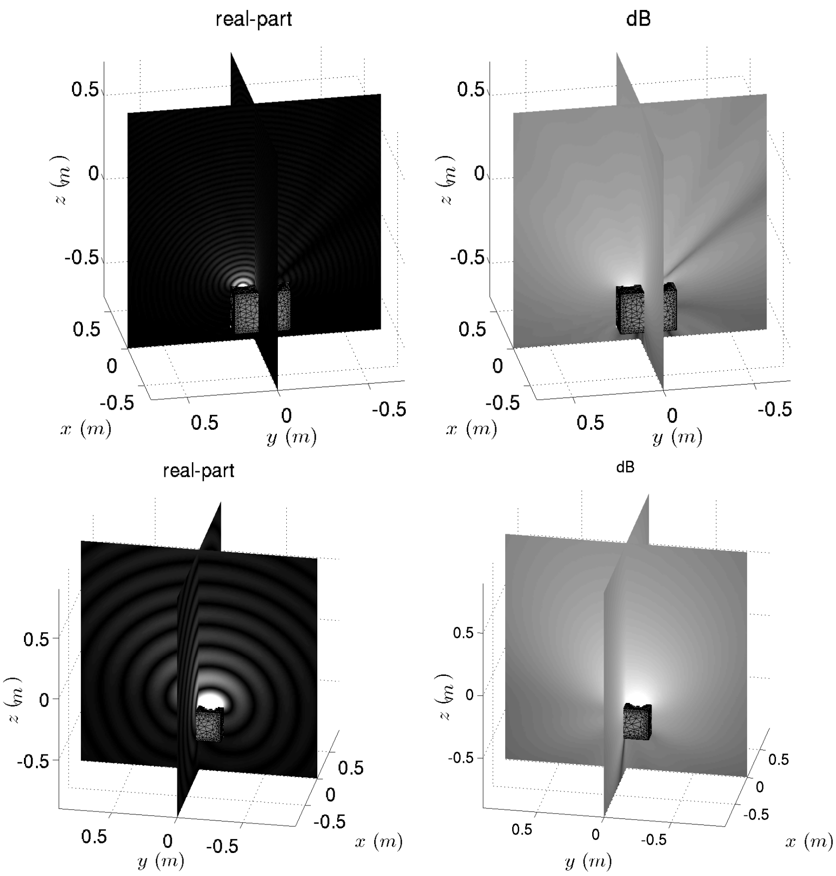


Figure 4: *Pressure fields. Top: For the discant element simulation at 5000 Hz. Bottom: For the bass element simulation at 1000 Hz.*

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