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MODELLING THE DIRECTIONAL CHARACTERISTICS OF SOUND REFLECTIONS

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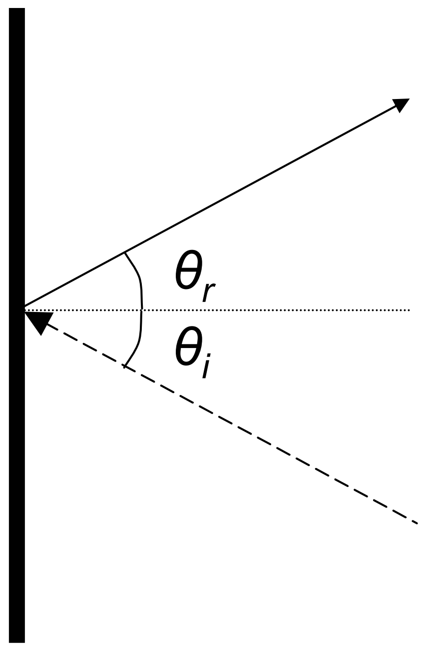
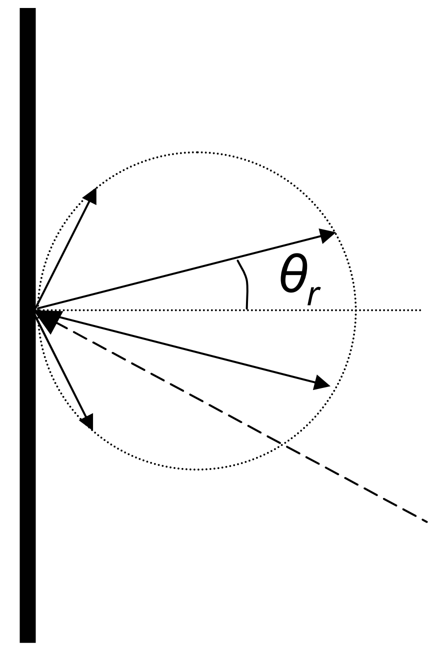
Introduction

- For room acoustic modelling is searched a better method to take the directional characteristics of sound reflections into account
- If successful, this may be introduced in the ODEON program

Reflection models (asymptotic models for short wavelengths)

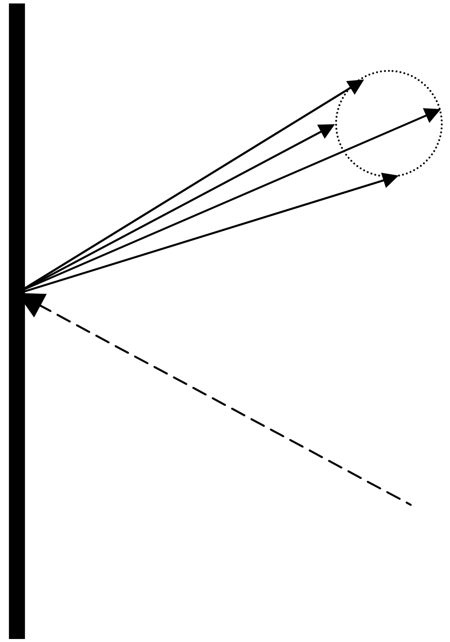




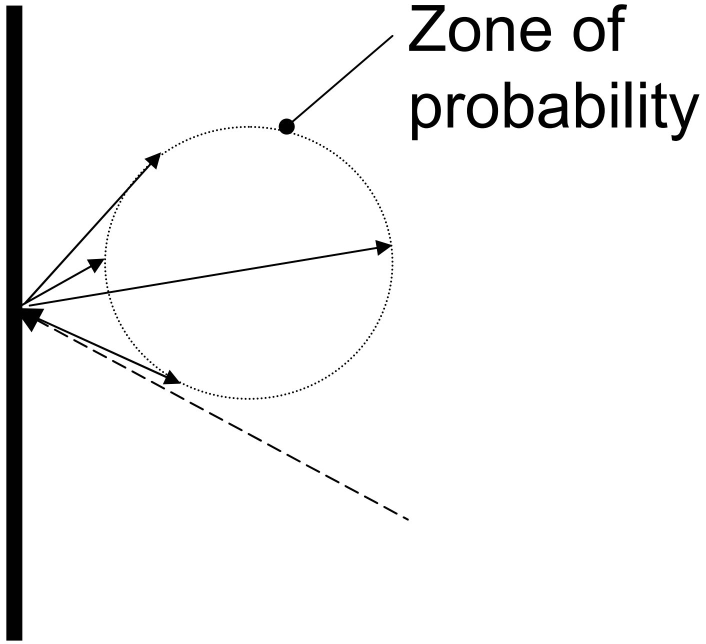
Snell's law: $\theta_r = \theta_i$

Lambert's law: Probability of diffuse reflection is ~ cos θ_r

Scattering as a weighted vector addition of specular and diffuse reflection

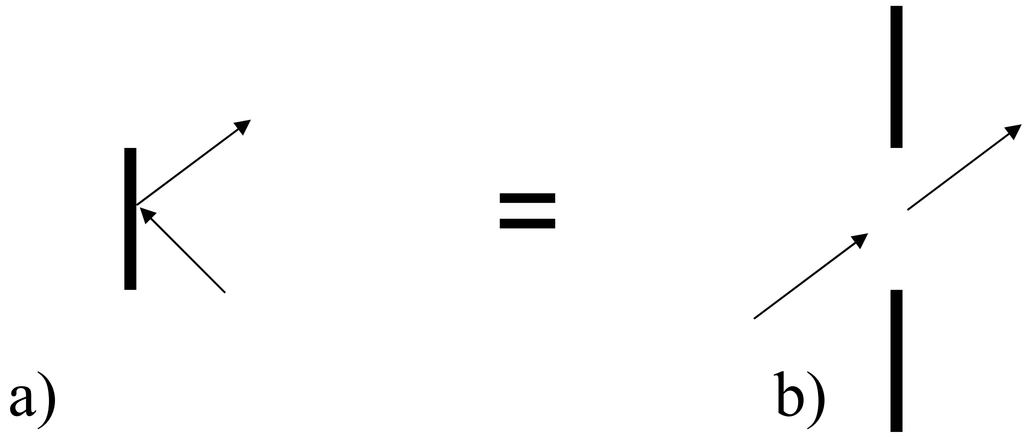


Small scattering,
s = 0,2



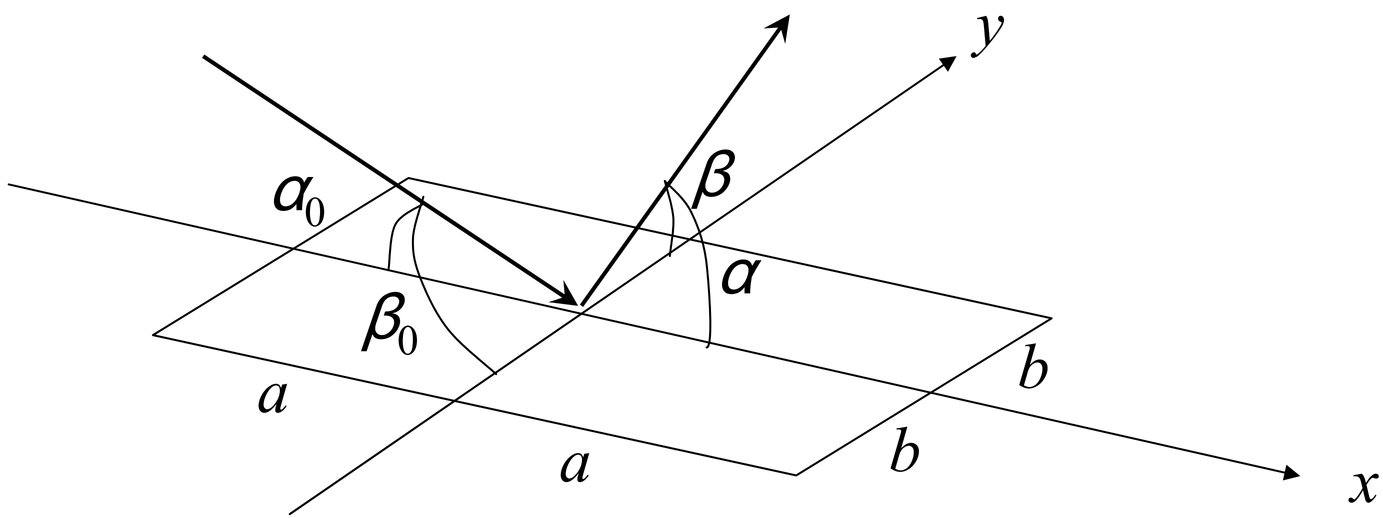
High scattering,
s = 0,8

Babinet’s Principle



Reflection from a surface (a) is equivalent to transmission through an aperture (b) with same size and shape, surrounded by a rigid baffle

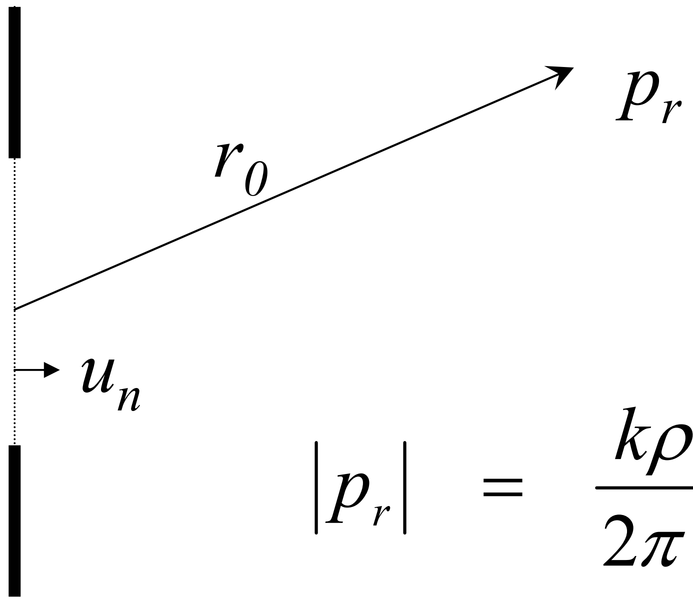
Definitions



$$u_n = |u_n| \cdot \exp\left(j\omega t - jkx\cos\alpha_0 - jky\cos\beta_0\right)$$

for $-a \le x \le a$ and $-b \le y \le b$

Solution from sound insulation theory



$$|p_r| = \frac{k\rho c}{2\pi r_0} \cdot |u_n| \cdot S \cdot \frac{\sin X}{X} \cdot \frac{\sin Y}{Y}$$

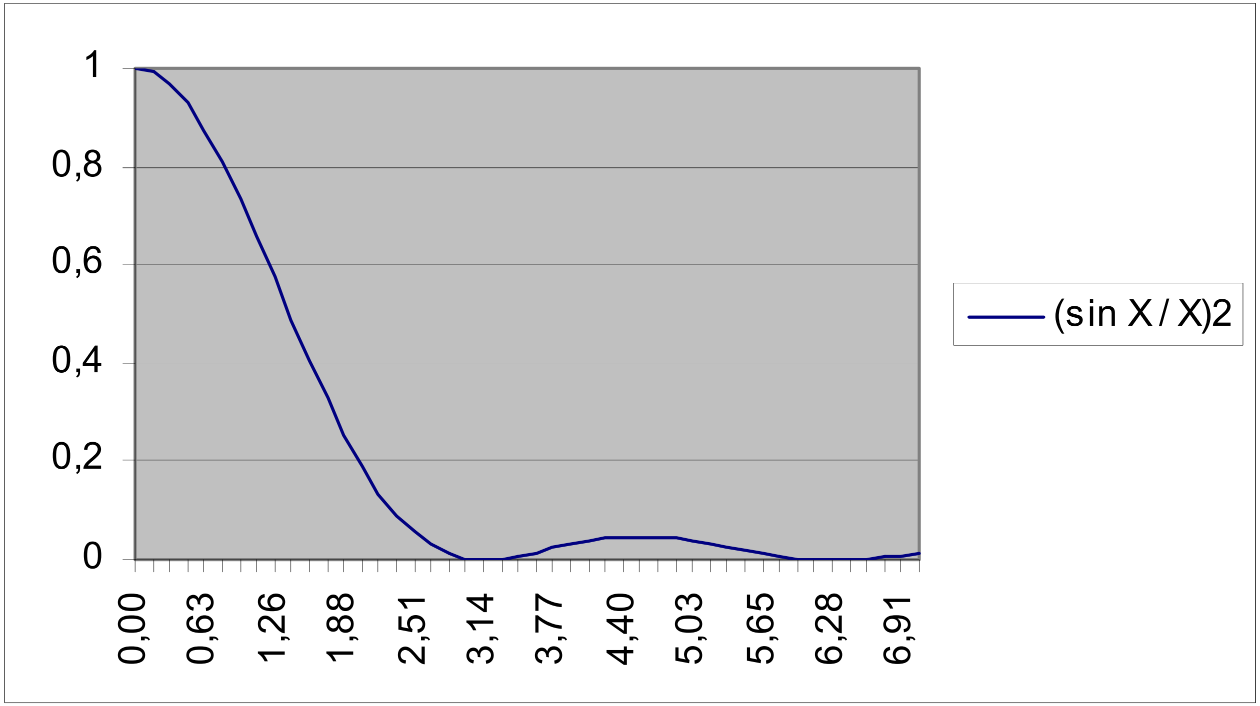
$$\text{Area}: \quad S = (2a)(2b)$$

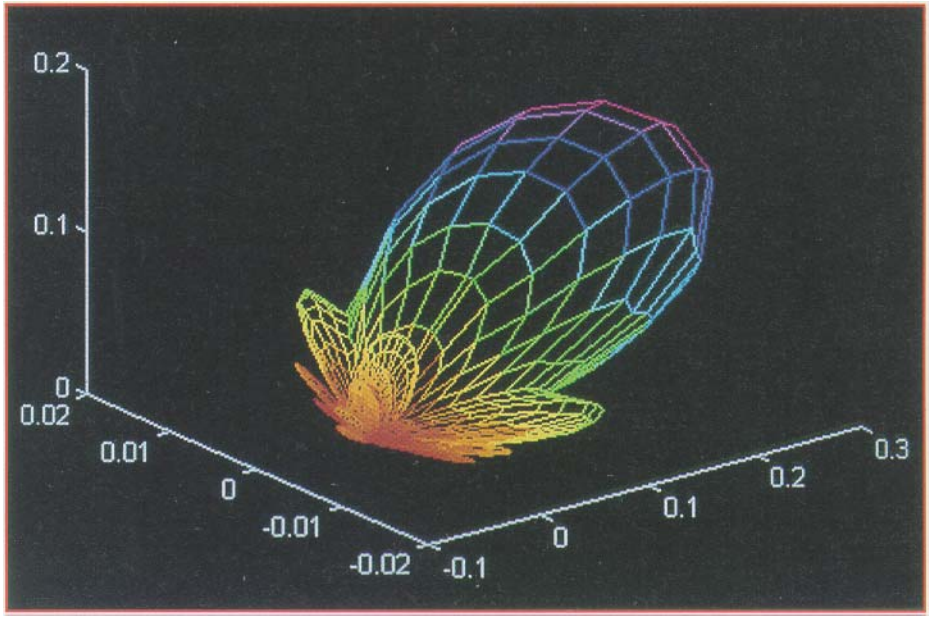
Radiation function for a single frequency

$$\frac{I_r}{I_{r,\max}} = \left(\frac{\sin X}{X} \cdot \frac{\sin Y}{Y} \right)^2$$

$$X = ka(\cos\alpha - \cos\alpha_0)$$
$$Y = kb(\cos\beta - \cos\beta_0)$$

Radiation function for a single frequency

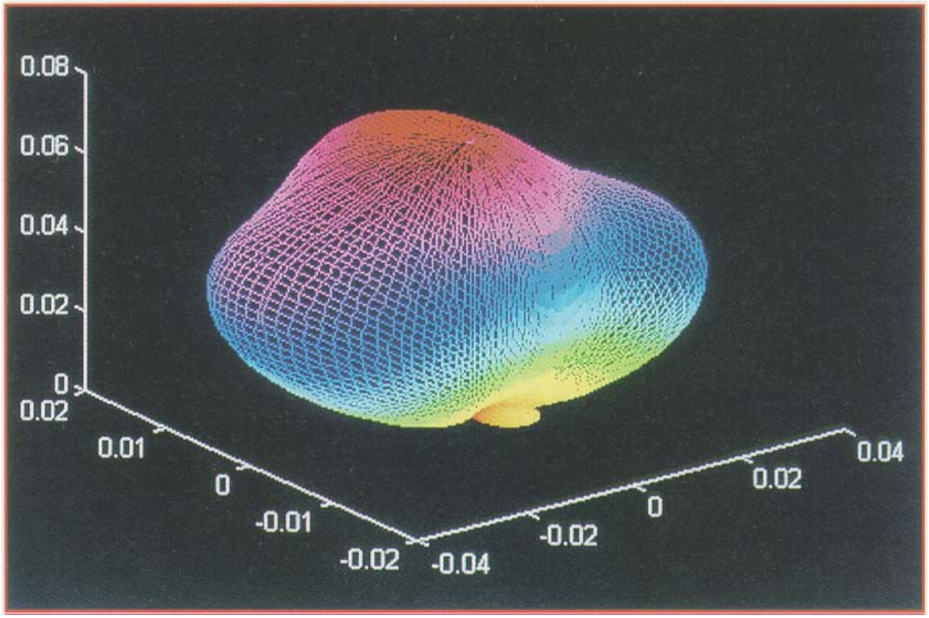


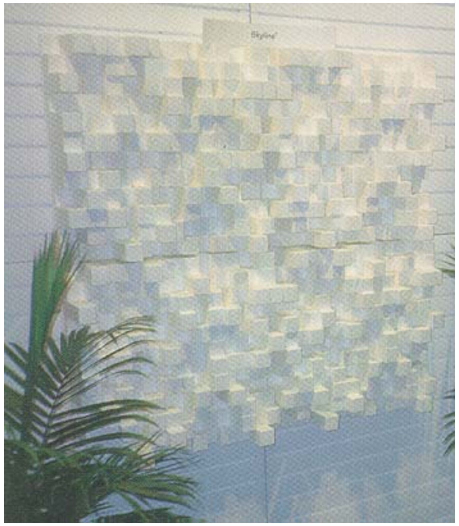


Measurements, M. Kleiner (1996)

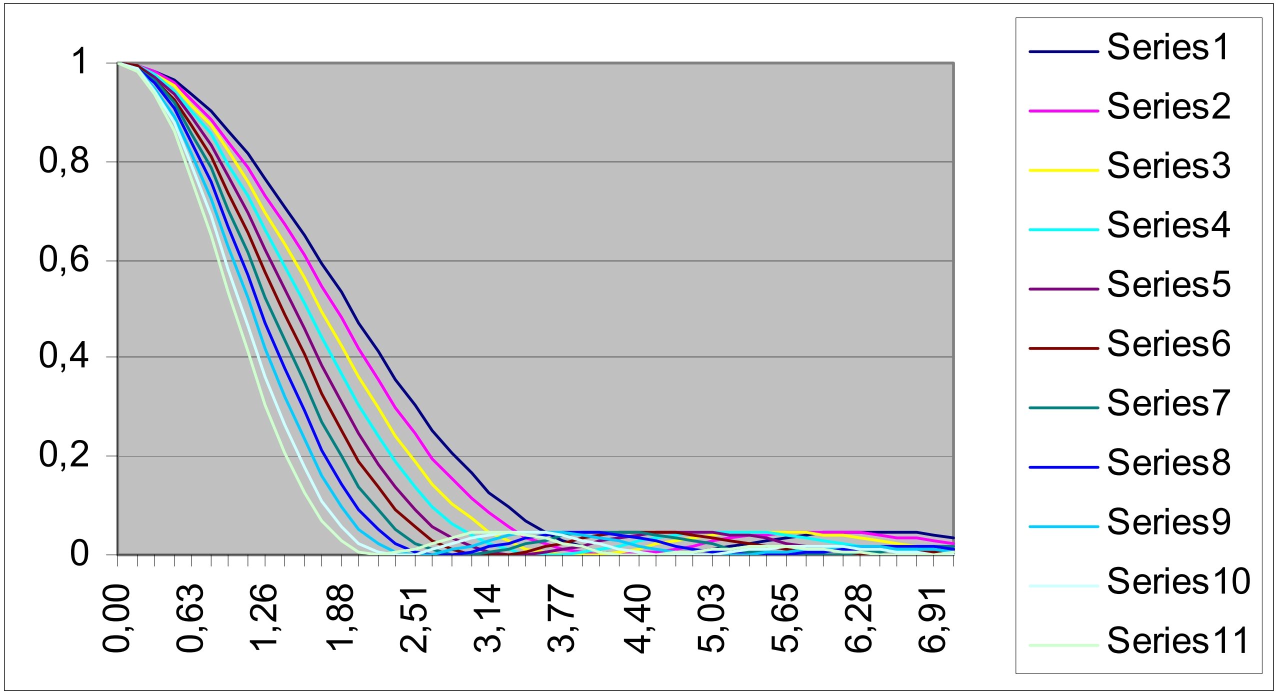
A: Plane surface

B: Diffusing surface

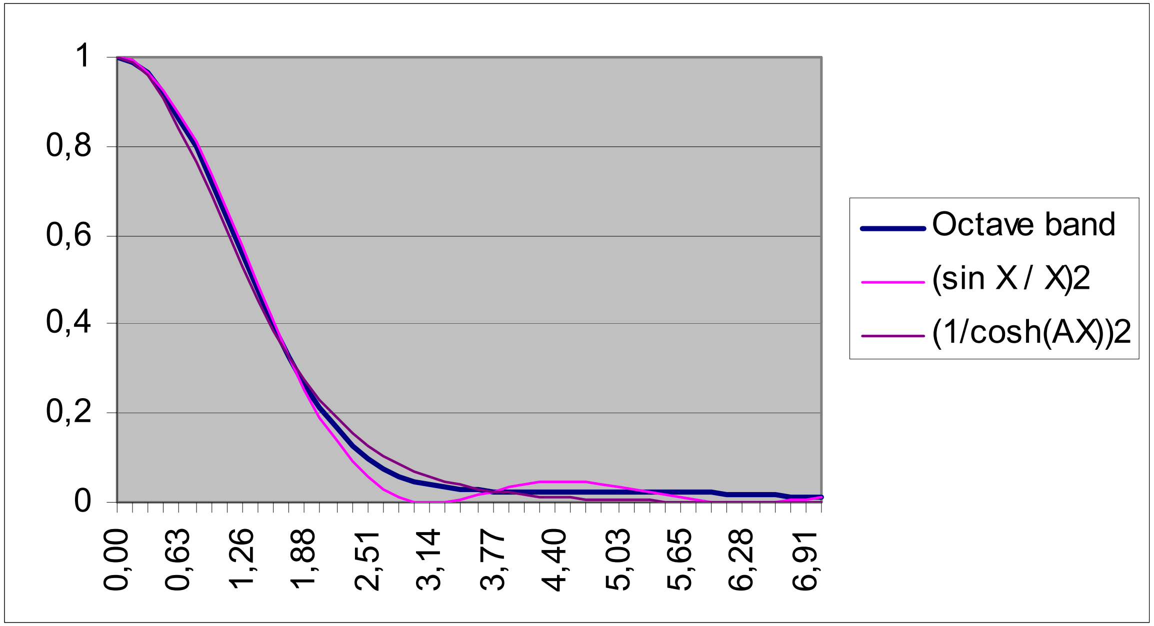




Variation of radiation function within an octave band

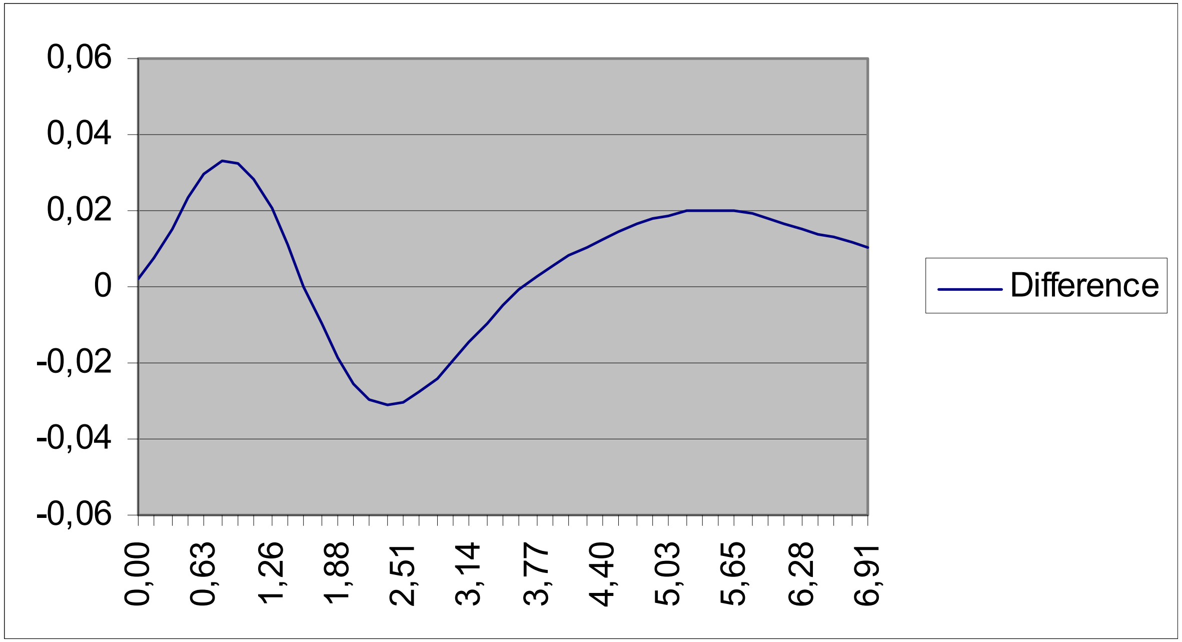


Radiation functions for an octave band, for the centre frequency, and the suggested approximation



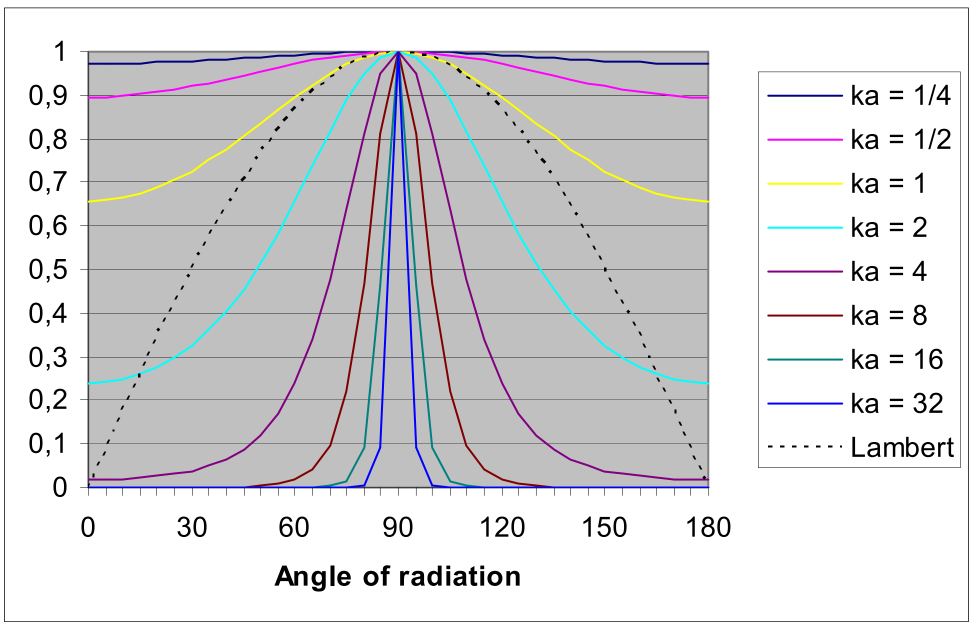
$$\left\langle \left(\frac{\sin X}{X} \right)^2 \right\rangle_{octave} \approx \left(\cosh(0.67 \cdot X) \right)^{-2}$$

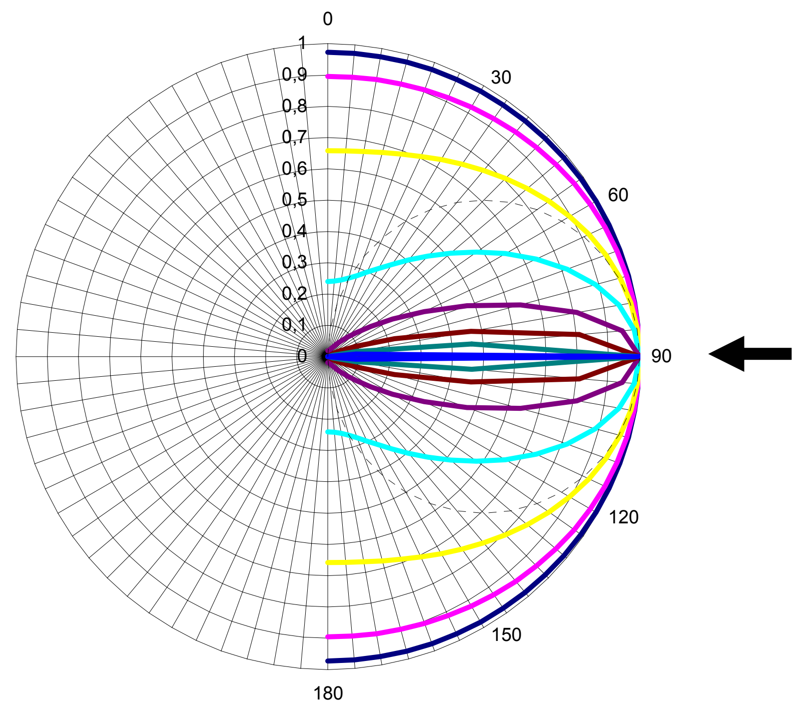
Difference between octave band average and the suggested approximation with the constant *A* = 0.67.



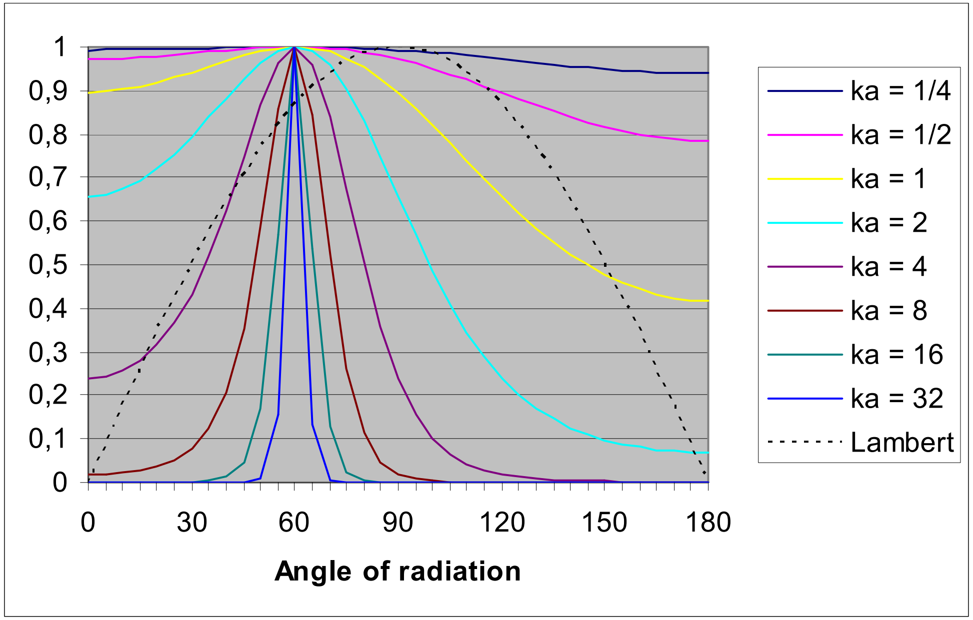
$$\left\langle \left(\frac{\sin X}{X} \right)^2 \right\rangle_{octave} = \left(\cosh(0.67 \cdot X)\right)^{-2} \pm \varepsilon \; ; \qquad \varepsilon \leq 0.03$$

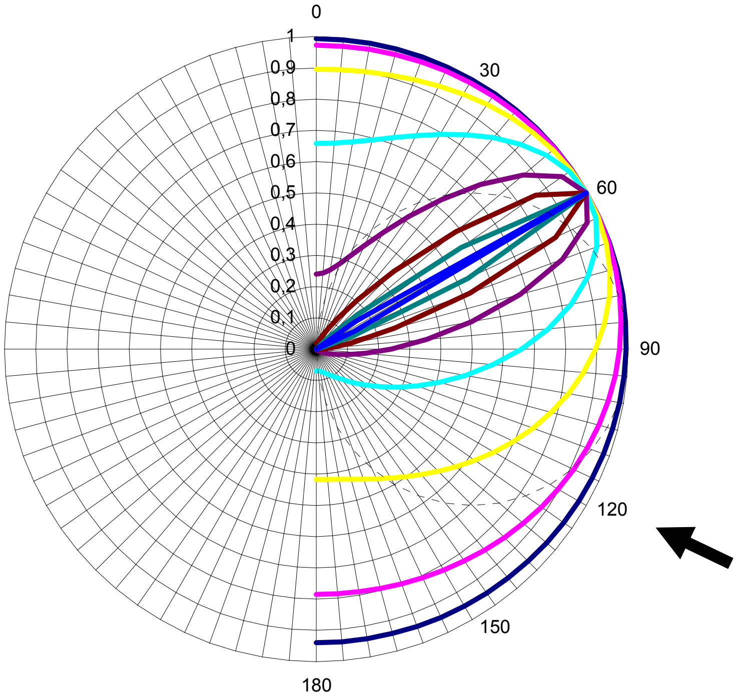
Directivity for angle of incidence α_0 = 90°



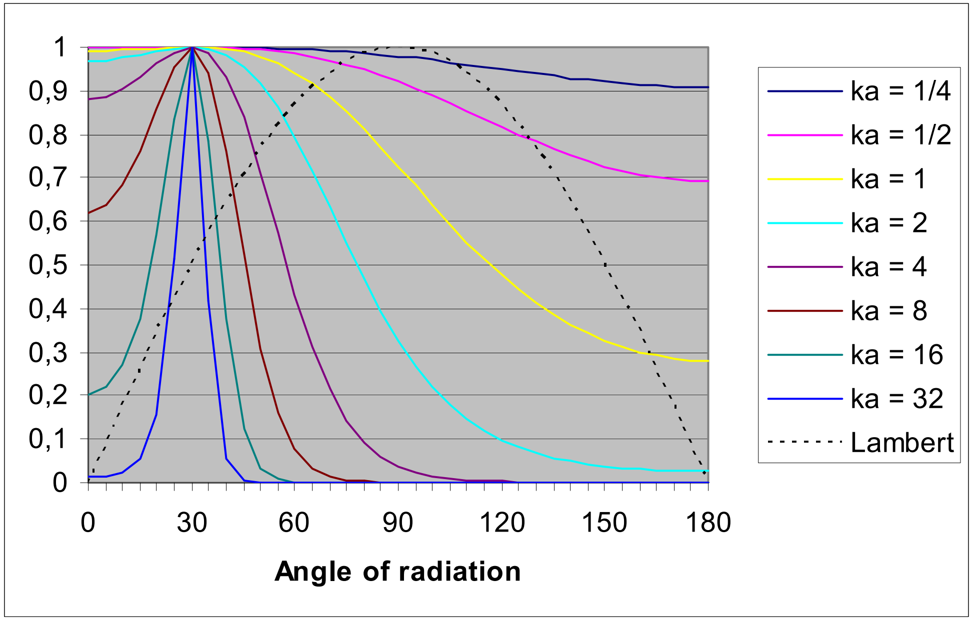


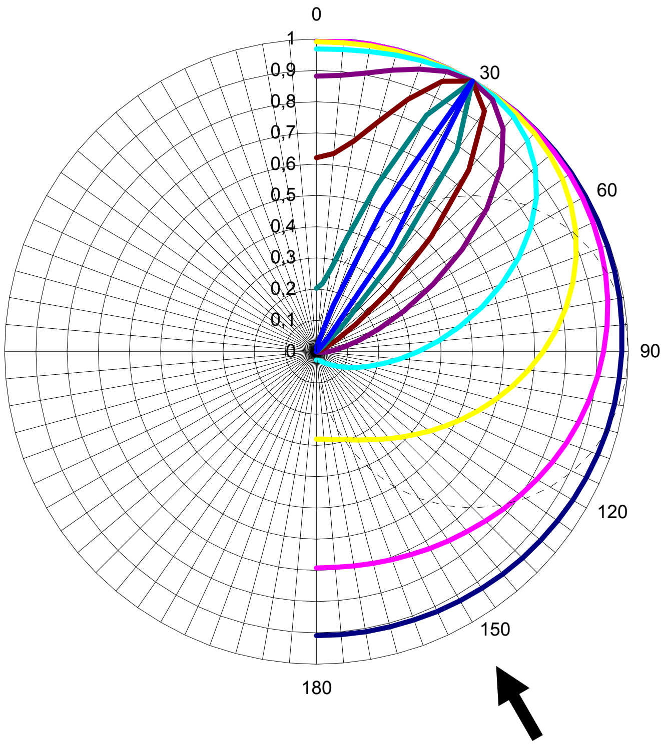
Directivity for angle of incidence $\alpha_0 = 60°$



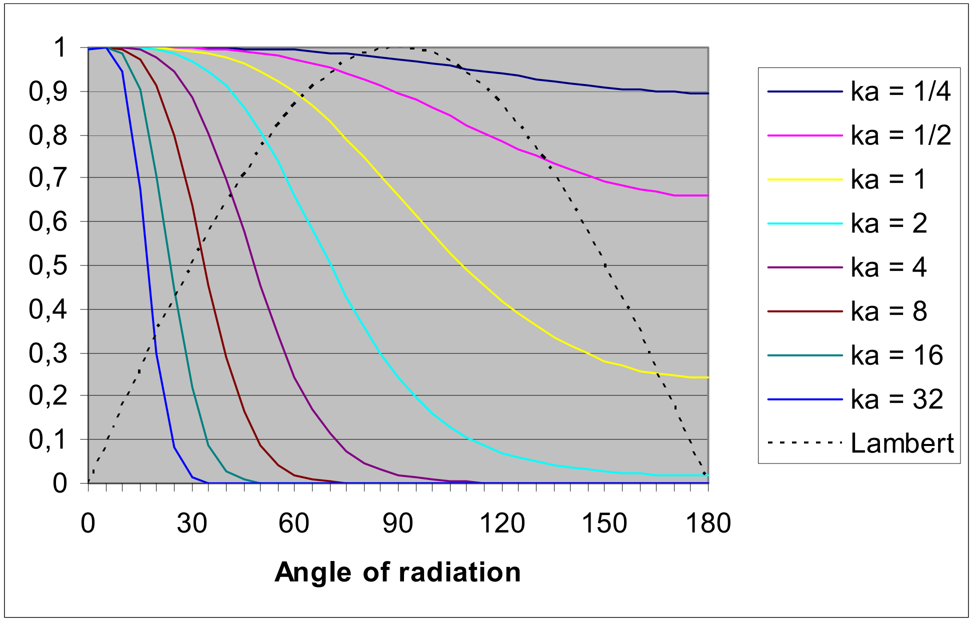


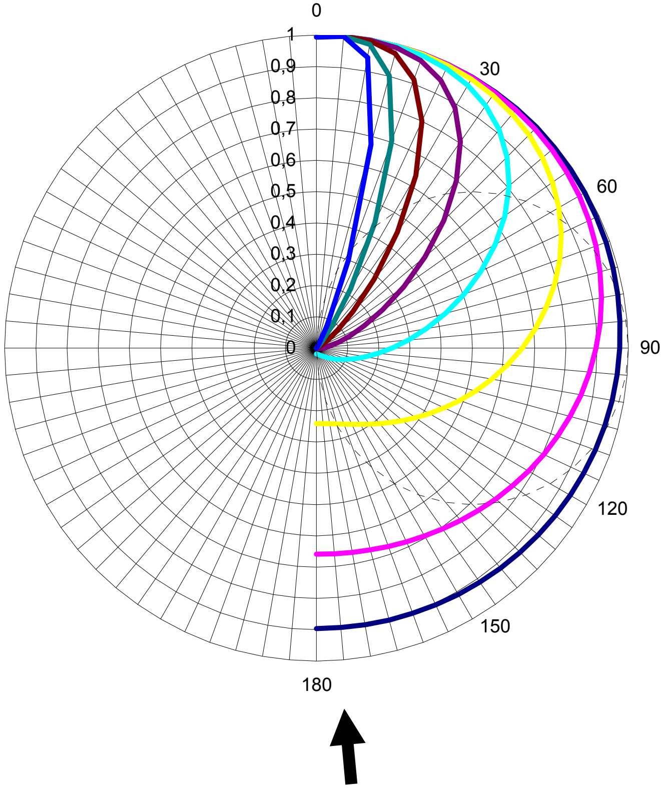
Directivity for angle of incidence α_0 = 30°





Directivity for angle of incidence α_0 = 5°





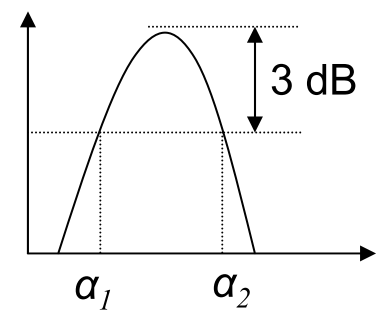
Examples of dimensions and frequencies

2a	*ka = 1/4*	*ka = 1/2*	*ka = 1*	*ka = 2*	*ka = 4*	*ka = 8*	*ka = 16*	*ka = 32*
0,22 m	125	250	500	1000	2000	4000	8000	
0,44 m	63	125	250	500	1000	2000	4000	8000
0,88 m		63	125	250	500	1000	2000	4000
1,75 m			63	125	250	500	1000	2000
3,50 m				63	125	250	500	1000

Application to a ray tracing model

Average distance between *N* rays in a distance r_1 from source

$$d \approx \sqrt{\frac{4\pi\, r_1^2}{N}} = 2\, r_1 \sqrt{\frac{\pi}{N}}$$

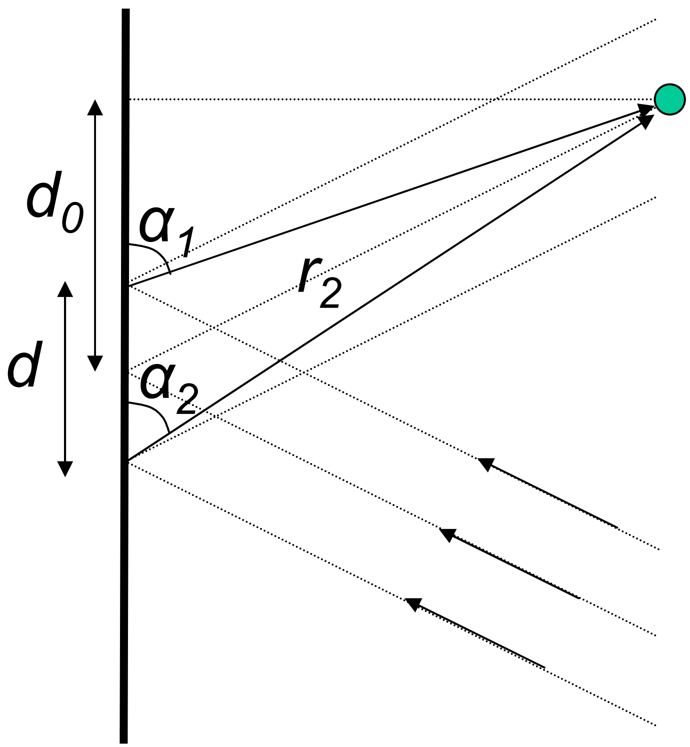


3 dB bandwidth of radiation function:

$$B = \alpha_2 - \alpha_1; \qquad \cosh\left(0.67 \cdot \left|X_{1,2}\right|\right) = \sqrt{2} \quad \Rightarrow \quad X_{1,2} = \pm 1.3155$$

$$X_{1,2} = ka(\cos\alpha_{1,2} - \cos\alpha_0) \quad \Rightarrow \quad \cos\alpha_{1,2} = \cos\alpha_0 \pm \frac{1.3155}{ka}$$

Two reflections meeting at the -3 dB point of the reflection directivity



$$\cos\alpha_1 \cong \left(d_0 - \frac{d}{2}\right)/r_2$$

$$\cos\alpha_2 \cong \left(d_0 + \frac{d}{2}\right)/r_2$$

$$\cos\alpha_2 - \cos\alpha_1 \cong d/r_2 = 2\frac{r_1}{r_2}\sqrt{\frac{N}{\pi}} = 2\frac{1.3155}{ka}$$

Application to a ray tracing model

The directivity function can be used in a ray tracing model below a certain frequency, which depends on the distance to source and receiver, number of rays and the size of the reflecting surface.

$$(ka)_{\max} \approx 1{,}3155 \cdot \frac{r_2}{r_1}\sqrt{\frac{N}{\pi}}$$

k : angular wave number
a : half length of reflecting surface
r_1 : distance from source to reflecting surface
r_2 : distance from reflecting surface to receiver
N : number of rays

Conclusion

- The known solution for sound transmission through a rectangular panel may be adopted for the sound reflection
- The directivity function for an octave band may be efficiently approximated by the function:

$$\left\langle \left(\frac{\sin X}{X} \right)^2 \right\rangle_{octave} \approx \left(\cosh(0.67 \cdot X) \right)^{-2}$$